

Calculus

Adopted 2023

Functions, Graphs, and Limits

Limits of Functions

- A. Understand the concept of the limit of a function. **C.F.LF.A**
 - 1. Calculate limits (including limits at infinity) using algebra. **C.F.LF.A.1**
 - 2. Estimate limits of functions (including one-sided limits) from graphs or tables of data. Apply the definition of a limit to a variety of functions, including piecewise functions. **C.F.LF.A.2**
 - 3. Draw a sketch that illustrates the definition of the limit; develop multiple real-world scenarios that illustrate the definition of the limit. **C.F.LF.A.3**
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Behavior of Functions

- A. Describe the asymptotic and unbounded behavior of functions. **C.F.BF.A**
 - 1. Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity. **C.F.BF.A.1**
 - 2. Discuss the various types of end behavior of functions; identify prototypical functions for each type of end behavior. **C.F.BF.A.2**
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Continuity

- A. Develop an understanding of understanding of continuity as a property of functions **C.F.C.A**
 - 1. Define continuity at a point using limits; define continuous functions. **C.F.C.A.1**
 - 2. Determine whether a given function is continuous at a specific point. **C.F.C.A.2**
 - 3. Determine and define different types of discontinuity (point, jump, infinite) in terms of limits. **C.F.C.A.3**
 - 4. Apply the Intermediate Value Theorem and Extreme Value Theorem to continuous functions. **C.F.C.A.4**
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Derivatives

Understand the Concept of the Derivative

- A. Demonstrate an understanding of the derivative. **C.D.CD.A**
 - 1. Represent and interpret the derivative of a function graphically, numerically, and analytically. **C.D.CD.A.1**
 - 2. Interpret the derivative as an instantaneous rate of change. **C.D.CD.A.2**
 - 3. Define the derivative as the limit of the difference quotient; illustrate with the sketch of a graph. **C.D.CD.A.3**
 - 4. Demonstrate the relationship between differentiability and continuity. **C.D.CD.A.4**
- B. Understand the derivative at a point. **C.D.CD.B**
 - 5. Interpret the derivative as the slope of a curve (which could be a line) at a point, including points at which there are vertical tangents and points at which there are no tangents (i.e., where a function is not locally linear). **C.D.CD.B.5**
 - 6. Approximate both the instantaneous rate of change and the average rate of change given a graph or table of values. **C.D.CD.B.6**
 - 7. Write the equation of the line tangent to a curve at a given point. **C.D.CD.B.7**
 - 8. Apply the Mean Value Theorem. **C.D.CD.B.8**
 - 9. Understand Rolle's Theorem as a special case of the Mean Value Theorem. **C.D.CD.B.9**

Computing and Applying Derivatives

A. Apply differentiation techniques. C.D.AD.A

1. Describe in detail how the basic derivative rules are used to differentiate a function; discuss the difference between using the limit definition of the derivative and using the derivative rules. C.D.AD.A.1
2. Calculate the derivative of basic functions (power, exponential, logarithmic, and trigonometric). C.D.AD.A.2
3. Calculate the derivatives of sums, products, and quotients of basic functions. C.D.AD.A.3
4. Apply the chain rule to find the derivative of a composite function. C.D.AD.A.4
5. Implicitly differentiate an equation in two or more variables C.D.AD.A.5
6. Use implicit differentiation to find the derivative of the inverse of a function. C.D.AD.A.6

B. Use first and second derivatives to analyze a function. C.D.AD.B

7. Relate the increasing and decreasing behavior of f to the sign of f' both analytically and graphically. C.D.AD.B.7
8. Use the first derivative to find extrema (local/relative and global/absolute). C.D.AD.B.8
9. Analytically locate the intervals on which a function is increasing, decreasing, or neither. C.D.AD.B.9
10. Relate the concavity of f to the sign of f'' both analytically and graphically. C.D.AD.B.10
11. Use the second derivative to find points of inflection as points where concavity changes. C.D.AD.B.11
12. Analytically locate intervals on which a function is concave up, concave down, or neither. C.D.AD.B.12
13. Relate corresponding characteristics of the graphs of f , f' , and f'' . C.D.AD.B.13
14. Translate verbal descriptions into equations involving derivatives and vice versa. C.D.AD.B.14

C. Apply derivatives to solve problems. C.D.AD.C

15. Model rates of change, including related rates problems. In each case, include a discussion of units. C.D.AD.C.15
 16. Solve optimization problems to find a desired maximum or minimum value. C.D.AD.C.16
 17. Use differentiation to solve problems involving velocity, speed, and acceleration. C.D.AD.C.17
 18. Use tangent lines to approximate function values and changes in function values when inputs change (linearization). C.D.AD.C.18
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Integrals

Understanding Integrals

- A. Demonstrate understanding of a definite integral. **C.I.UI.A**
 - 1. Define the definite integral as the limit of Riemann sums and as the net accumulation of change. **C.I.UI.A.1**
 - 2. Write a Riemann sum that represents the definition of a definite integral. **C.I.UI.A.2**
 - 3. Use Riemann sums (left, right, and midpoint evaluation points) and trapezoid sums to approximate definite integrals of functions represented graphically, numerically, and by tables of values. **C.I.UI.A.3**
 - B. Understand and apply the Fundamental Theorem of Calculus. **C.I.UI.B**
 - 4. Recognize differentiation and antidifferentiation as inverse operations. **C.I.UI.B.4**
 - 5. Evaluate definite integrals using the Fundamental Theorem of Calculus. **C.I.UI.B.5**
 - 6. Use the Fundamental Theorem of Calculus to represent a particular antiderivative of a function and to understand when the antiderivative so represented is continuous and differentiable. **C.I.UI.B.6**
 - 7. Apply basic properties of definite integrals (e.g. additive, constant multiple, translations). **C.I.UI.B.7**
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Calculate and Apply Integrals

- A. Apply techniques of antidifferentiation. **C.I.AI.A**
 - 1. Find antiderivatives that follow directly from derivatives of basic functions (power, exponential, logarithmic, and trigonometric). **C.I.AI.A.1**
 - 2. Use substitution of variables to calculate antiderivatives (including changing limits for definite integrals). **C.I.AI.A.2**
 - 3. Find specific antiderivatives using initial conditions. **C.I.AI.A.3**
- B. Apply integrals to solve problems. **C.I.AI.B**
 - 4. Use a definite integral to find the area of a region. **C.I.AI.B.4**
 - 5. Use a definite integral to find the volume of a solid formed by rotating a region around a given axis. **C.I.AI.B.5**
 - 6. Use integrals to solve a variety of problems (e.g., distance traveled by a particle along a line, exponential growth/decay). **C.I.AI.B.6**