South Dakota Mathematics

Grades 9, 10, 11, 12

Adopted 2018

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.** MP.1
2. **Reason abstractly and quantitatively.** MP.2
3. **Construct viable arguments and critique the reasoning of others.** MP.3
4. **Model with mathematics.** MP.4
5. **Use appropriate tools strategically.** MP.5
6. **Attend to precision.** MP.6
7. **Look for and make use of structure.** MP.7
8. **Look for and express regularity in repeated reasoning.** MP.8

High School - Algebra I

The Real Number System

Extend the properties of exponents to rational exponents.

1. Explain how the definition of rational exponents follows from extending the properties of integer exponents, allowing for a notation for radicals in terms of rational exponents. N.RN.1
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. N.RN.2

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; the sum of a rational and an irrational number is irrational; and the product of a nonzero rational and an irrational number is irrational. N.RN.3

Quantities

Reason quantitatively and use units to solve problems.

1. Use unit analysis to understand and guide the process of solving multi-step problems; choose and interpret units consistently in formulas; and choose and interpret the scale and origin in graphs and data displays. N.Q.1
2. Define appropriate quantities for the purpose of descriptive modeling. N.Q.2
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. N.Q.3

Seeing Structure in Expression

Interpret the structure of expressions.

i. Interpret expressions that represent a quantity in terms of its context. A.SSE.1.I
   a. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1.I.A
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity in context. A.SSE.1.I.B

i. Recognize and use the structure of an expression to identify ways to rewrite it. A.SSE.2.I

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.3
   a. Factor a quadratic expression to reveal the zeros of the function it defines. A.SSE.3.A
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. A.SSE.3.B
   c. Use the properties of exponents to write equivalent expressions for exponential functions. A.SSE.3.C

Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A.APR.1

Creating Equations

Create equations that describe numbers or relationships.

i. Create equations and inequalities in one variable arising from situations in which linear, quadratic, and exponential functions are appropriate and use them to solve problems. A.CED.1.I

i. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.2.I

i. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. A.CED.3.I

i. Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations. A.CED.4.I

Reasoning With Equations and Inequalities

Understand solving equations as a process of reasoning.

1. Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A.REI.1

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. A.REI.3

i. Solve quadratic equations in one variable. A.REI.4.I

   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. A.REI.4.A

   b. Derive the quadratic formula from this form completing the square. A.REI.4.B

   c. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. A.REI.4.C

Solve systems of equations.

5. Understand the principles of the elimination method. A.REI.5

6. Solve systems of linear equations exactly and approximately by graphing, focusing on pairs of linear equations in two variables. A.REI.6

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. A.REI.7

Represent and solve equations and inequalities graphically.

i. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.10.I

i. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, including but not limited to using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic and exponential. A.REI.11.I

12. Graph a linear inequality (strict or inclusive) in two variables; graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. A.REI.12

Interpreting Functions

Understand the concept of a function and use functions notation.

1. Understand that a function maps each element of the domain to exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph off is the graph of the equation y = f(x). F.IF.1

2. Use function notation, evaluate functions, and interpret statements that use function notation in terms of a context. F.IF.2

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. F.IF.3

Interpret functions that arise in applications in terms of the context.

i. For functions, including linear, quadratic, and exponential, that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.4.I

i. Relate the domain of a function to its graph and find an appropriate domain in the context of the problem. F.IF.5.I

6. Calculate and interpret the average rate of change of a function, both symbolically and from a table over a specified interval. Estimate the rate of change from a graph. F.IF.6

Analyze functions using different representations.

i. Graph parent functions and their transformations expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.7.I

   a. Graph linear, exponential, and quadratic functions and show intercepts, maxima, and minima. F.IF.7.I.A

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.8

   a. Use the process of graphing, factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. F.IF.8.A

   b. Interpret expressions for exponential growth and decay. F.IF.8.B

i. Compare properties of two functions (linear, quadratic and exponential) each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). F.IF.9.I

Building Functions

Build a function that models a relationship between two quantities.

i. Write a function (linear, quadratic, and exponential) that describes a relationship between two quantities. F.BF.1.I

   a. Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.1.I.A

   b. Determine an explicit expression from a graph. F.BF.1.I.B

   c. Combine standard function types using arithmetic operations. F.BF.1.I.C

2. Write arithmetic and geometric sequences both recursively and with an explicit formula and use them to model situations. F.BF.2

Build new functions from existing functions.

i. Identify the effect on the graph of f(x) (linear, exponential, quadratic) replaced with f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with contrasting cases and illustrate an explanation of the effects on the graph using technology. F.BF.3.I

Linear, Quadratic and Exponential Models

Construct and compare linear and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. F.LE.1

   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. F.LE.1.A

   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. F.LE.1.B

   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.1.C

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.2

3. Recognize, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. F.LE.3

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context. F.LE.5

Interpreting Categorical and Quantitative Data

Summarize, represent and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots). S.ID.1
2. Use statistics appropriate to the shape and context of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.2
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). S.ID.3

Summarize, represent and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.ID.5
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. S.ID.6
    a. Determine the function (linear, quadratic, or exponential model) that best fits a set of data and use that function fitted to data to solve problems within context. S.ID.6.A
    b. Informally and using technology assess the fit of a function by plotting and analyzing residuals. S.ID.6.B
    c. Fit a linear function for a scatter plot that suggests a linear association. S.ID.6.C

Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.6.7
8. Compute (using technology) and interpret the correlation coefficient of a linear fit. S.ID.6.8
9. Distinguish between correlation and causation. S.ID.6.9

High School - Geometry **Congruence**

Experiment with transformations in the plane.

1. State and apply precise definitions of angle, circle, perpendicular, parallel, ray, line segment, and distance based on the undefined notions of point, line, and plane. G-CO.1
2. Represent transformations in the plane. (e.g., using transparencies and/or geometry software); G-CO.2
   a. Describe transformations as functions that take points in the plane as inputs and give other points as outputs. G-CO.2.A
   b. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus dilation). G-CO.2.B
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and/or reflections that map the figure onto itself. G-CO.3
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G-CO.4
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure, (e.g., using graph paper, tracing paper, or geometry software). Specify a sequence of transformations that will map a given figure onto another. G-CO.5

Understand congruence in terms of rigid motions.

6. Use geometric descriptions of rigid motions to transform figures. G-CO.6
   a. Predict the effect of a given rigid motion on a given figure. G-CO.6.A
   b. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G-CO.6.B
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G-CO.7
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. G-CO.8

Prove geometric theorems.

9. Prove theorems about lines and angles. Theorems must include but not limited to: vertical angles are congruent; when a transversal intersects parallel lines, alternate interior angles are congruent and same side interior angles are supplementary (using corresponding angles postulate); points on a perpendicular bisector of a line segment are equidistant from the segment's endpoints. G-CO.9
10. Prove congruence theorems about triangles. Theorems must include but not limited to: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the mid segment of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. G-CO.10

11. Prove theorems about parallelograms. Theorems must include but not limited to: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. G-CO.11

Make geometric constructions.

12. Perform geometric constructions with a compass and straightedge. including copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines/segments, constructing a line parallel to a given line through a point not on the line. G-CO.12
13. Construct an equilateral triangle, a square, and a regular hexagon. G-CO.13

Similarity, Right Triangles and Trigonometry

Understand similarity in terms of similarity transformations.

1. Verify experimentally and apply the properties of dilations as determined by a center and a scale factor. G-SRT.1
2. Determine whether figures are similar, using the definition of similarity and using similarity transformations. G-SRT.2
3. Use the properties of similarity transformations to establish similarity theorems. Theorems must include AA, SAS, and SSS. G-SRT.3

Prove theorems involving similarity.

4. Prove theorems about triangles involving similarity. Theorems must include but not limited to: a line parallel to one side of a triangle divides the other two proportionally, and its converse; the Pythagorean Theorem proved using triangle similarity. G-SRT.4
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. G-SRT.5

Define trigonometric ratios and solve problems involving right triangles.

6. Define, using similarity, that side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios (sine, cosine, and tangent) for acute angles. G-SRT.6
7. Explain and use the relationship between the sine and cosine of complementary angles. G-SRT.7
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. G-SRT.8

Circles

Understand and apply theorems about circles.

1. Prove that all circles are similar. G-C.1
2. Identify and describe relationships among central angles, inscribed angles, circumscribed angles, radii, and chords. G-C.2
3. Construct, using a compass and straight edge, the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G-C.3
4. Construct a tangent line from a point outside a given circle to the circle. G-C.4

Find arc lengths and areas of sectors of circles.

5. Derive using similarity the length of the arc intercepted by an angle is proportional to the radius. G-C.5
    a. Define the radian measure of the angle as the constant of proportionality; G-C.5.A
    b. Derive and apply the formula for the area of a sector. G-C.5.B

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G-GPE.1
2. Use coordinates to prove geometric relationships algebraically. For example, determine whether a figure defined by four given points in the coordinate plane is a rectangle; determine whether the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2). G-GPE.2
3. Define and use the slope criteria for parallel and perpendicular lines. (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G-GPE.3

Use coordinates to prove simple geometric theorems algebraically.

4. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. e.g. Determine the point(s) that divide the segment with endpoints of (-4, 7) and (6, 3) into the ratio 2:3 G-GPE.4
5. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. G-GPE.5

Geometric Measurement and Dimension

Explain volume and surface area formulas and use them to solve problems.

1. Give an informal argument for the formulas for the volume of a cylinder, pyramid, sphere, and cone. Use dissection arguments, and informal limit arguments. G-GMD.1
2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures G-GMD.2
3. Know and apply volume and surface area formulas for cylinders, pyramids, cones, and spheres for composite figures to solve problems. G-GMD.3

Visualize relationships between two-dimensional and three- dimensional objects.

4. Identify two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. G-GMD.4

Modeling with Geometry

Applying geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). G-MG.1
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). G-MG.2
3. Apply geometric concepts to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). G-MG.3

Statistics and Probability- Conditions Probability and Rules of Probability

Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space or as unions, intersections, or complements of other events. S-CP.1
2. Determine whether two events A and B are independent. S-CP.2
3. Determine conditional probabilities and interpret independence by analyzing conditional probability. S-CP.3
4. Construct and interpret two-way frequency tables of data. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. S-CP.4
5. Recognize and explain the concepts of conditional probability and independence in everyday language and situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. S-CP.5
6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the result. S-CP.6
7. Apply the Addition Rule, P(A or B), and interpret the result. S-CP.7
8. Apply the general Multiplication Rule, P(A and B), and interpret the result. S-CP.8

High School - Algebra II

Seeing Structure in Expressions

Interpret the structure of expressions.

ii. Interpret expressions that represent a quantity in terms of its context. A.SSE.1.II

   a. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1.II.A

   b. Interpret complicated expressions by viewing one or more of their parts as a single entity in context. A.SSE.1.II.B

ii. Recognize and use the structure of an expression to identify ways to rewrite it. A.SSE.2.II

Arithmetic with Polynomials and Rational Expressions

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem. A.APR.2
3. Identify zeros of polynomials by factoring. A.APR.3
   a. When suitable factorizations are available, use the zeros to construct a rough graph of the related function. A.APR.3.A
   b. When given a graph, use the zeros to construct a possible factorization of a polynomial. A.APR.3.B

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; using inspection, synthetic division, long division, box method or, for the more complicated examples, a computer algebra system. A.APR.6

Creating Equations

Create equations that describe numbers or relationships.

ii. Create equations and inequalities in one variable and use them to solve problems. A.CED.1.II

ii. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.2.II

ii. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. A.CED.3.II

ii. Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations. A.CED.4.II

Reasoning with Equations and Inequalities

Understand solving equations as a process of reason and explain the reasoning.

2. Solve rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. Rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2. Radical functions are limited to square roots or cube roots of at most quadratic polynomials. A.REI.2

Solve equations and inequalities in one variable.

ii. Select, justify and apply appropriate methods to solve quadratic equations in one variable. Recognize complex solutions and write them as a +/- bi for real numbers a and b. A.REI.4.II

Represent and solve equations and inequalities graphically.

ii. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, including but not limited to using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. A.REI.11.II

Interpreting Functions

Interpret functions that arise in applications in terms of the context.

ii. For functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries (including even, odd, or neither); end behavior; and periodicity. F.IF.4.II

ii. Relate the domain of a function to its graph and find an appropriate domain in the context of the problem. F.IF.5.II

Analyze functions using different representations.

ii. Graph parent functions and their transformations expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.7.II

   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.7.II.B

   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. F.IF.7.II.C

   d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. F.IF.7.II.D

   e. Graph logarithmic functions, showing intercepts and end behavior. F.IF.7.II.E

   f. Graph trigonometric functions (sine and cosine), showing period, midline, and amplitude. F.IF.7.II.F

ii. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). F.IF.9.II

Building Functions

Build new functions from existing functions.

ii. Write a function that describes a relationship between two quantities. F.BF.1.II

   a. Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.1.II.A

   b. Determine an explicit expression from a graph. F.BF.1.II.B

   c. Combine standard function types using arithmetic operations. F.BF.1.II.C

   d. Compose functions. F.BF.1.II.D

ii. Identify the effect on the graph of f(x) replaced with f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with contrasting cases and illustrate an explanation of the effects on the graph using technology. F.BF.3.II

ii. Find inverse functions. F.BF.4.II

   a. Solve an equation for the independent variable of a function f that has an inverse function and write an expression for the inverse. F.BF.4.II.A

   b. Verify by composition that one function is the inverse of another. F.BF.4.II.B

   c. Read values of an inverse function from a graph or a table, given that the function has an inverse. F.BF.4.II.C

ii. (+)Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. F.BF.5.II

Linear, Quadratic and Exponential Models

Construct and compare linear and exponential models and solve

4. For exponential models, express as a logarithm the solution to ab^(ct) = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. F.LE.4

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.1
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions (sine and cosine) to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.2

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions (sine and cosine) to model periodic phenomena with specified amplitude, frequency, and midline. F.TF.5

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(A) + \cos^2(A) = 1$ and use it to calculate trigonometric ratios. F.TF.8

The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi where a and b are real numbers. N.CN.1
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. N.CN.2

Use complex numbers in polynomials identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions. N.CN.7

Interpreting Categorical and Quantitative Data

Summarize, represent and interpret data on a single count or measurement variable.

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. S.ID.4

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. S.IC.1
2. Determine whether a specified model is consistent with results from a given data-generating process. S.IC.2

Make interferences and justify conclusions from sample surveys, experiments and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.3
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S.IC.4
5. Use data from a randomized experiment to compare two treatment groups; use simulations to decide if differences between parameters are significant. S.IC.5
6. Evaluate reports based on data. S.IC.6

High School - 4th Year Mathematics

The Complex Number System

Perform arithmetic operations with complex numbers.

3. (+)Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. N.CN.3

Represent complex numbers and their operations on the complex plane.

4. (+)Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. N.CN.4
5. (+)Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. N.CN.5
6. (+)Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. N.CN.6
8. (+)Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. N.CN.8
9. (+)Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. N.CN.9

Vector and Matrix Quantities

Represent and model with vector quantities.

1. (+)Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes N.VM.1
2. (+)Write a vector in component form. N.VM.2
3. (+)Solve problems involving velocity and other quantities that can be represented by vectors. N.VM.3

Perform operations on vectors.

4. (+)Add and subtract vectors. N.VM.4
    a. (+)Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. N.VM.4.A
    b. (+)Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. N.VM.4.B
    c. (+)Understand vector subtraction $v – w$ as $v + (–w)$, where $–w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. N.VM.4.C
5. (+)Multiply a vector by a scalar. N.VM.5
    a. (+)Represent scalar multiplication graphically by scaling vectors and/or reversing their direction; perform scalar multiplication component-wise. N.VM.5.A
    b. (+)Compute the magnitude of a scalar multiple cv. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$). N.VM.5.B

Perform operations on matrices and use matrices in applications.

6. (+)Use matrices to represent and manipulate data. N.VM.6
7. (+)Multiply matrices by scalars to produce new matrices. N.VM.7
8. (+)Add, subtract, and multiply matrices of appropriate dimensions. N.VM.8
9. (+)Understand that, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. N.VM.9
10. (+)Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. Discover that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. N.VM.10
11. (+)Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. N.VM.11

12. (+)Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. N.VM.12

Arithmetic with Polynomials and Rational Expressions

Use polynomial identities to solve problems.

5. (+)Know and apply the Binomial Theorem for the expansion of (x + y) ⁿ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. A.APR.5

Rewrite rational expressions.

7. (+)Discover that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. A.APR.7

Reasoning with Equations and Inequalities

Solve systems of equations.

8. (+)Represent a system of linear equations as a single matrix equation in a vector variable. A.REI.8
9. (+)Use matrices to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). A.REI.9
10. (+)Solve linear, quadratic, polynomial, and rational inequalities in two variables algebraically and graphically. A.REI.10

Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

4. (+)Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. A.SSE.4
5. (+)Use summation notation to describe the sums in a series. A.SSE.5

Interpreting Functions

Analyze functions using different representations.

7. (+)Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.7
    d. (+)Graph rational functions, identify zeros and vertical, horizontal, and slant asymptotes, and determine end behavior. F.IF.7.D
    e. (+)Graph exponential and logarithmic functions, showing relationships, intercepts and end behavior. F.IF.7.E
    f. (+)Graph all trigonometric functions, showing key features and applying transformations. F.IF.7.F

Building Functions

Build a function that models a relationship between two quantities.

1. (+)Write a function that describes a relationship between two quantities. F.BF.1
   c. (+)Compose functions in context. F.BF.1.C
4. (+)Find inverse functions. F.BF.4
   b. (+)Verify by composition that one function is the inverse of another. F.BF.4.B
   c. (+)Read values of an inverse function from a graph or a table, given that the function has an inverse. F.BF.4.C
   d. (+)Produce an invertible function from a non-invertible function by restricting the domain. F.BF.4.D
5. (+)Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. F.BF.5
6. (+)Use reciprocal properties to develop definitions for cotangent, cosecant, and secant. F.BF.6

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

3. (+)Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for π-x, π+x, and 2π–x in terms of their values for x, where x is any real number. F.TF.3
4. (+)Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. F.TF.4

Model periodic phenomena with trigonometric functions.

6. (+)Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F.TF.6
7. (+)Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. F.TF.7

Prove and apply trigonometric identities.

9. (+)Prove the addition and subtraction, half-angle, and double-angle formulas for sine, cosine, and tangent and use them to solve problems. F.TF.9
10. (+)Use fundamental trigonometric identities. F.TF.10
    a. (+)Verify trigonometric identities F.TF.10.A
    b. (+)Evaluate trigonometric functions F.TF.10.B
    c. (+)Write equivalent trigonometric expressions F.TF.10.C
    d. (+)Solve trigonometric equations. F.TF.10.D

Similarity, Right Triangles and Trigonometry

Apply trigonometry to general triangles.

9. (+)Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side and use the formula to solve problems. G.SRT.9

10. (+)Prove the Laws of Sines and Cosines and use them to solve problems involving right and non-right triangles. G.SRT.10

Expressing Geometric Properties

Translate between the geometric description and the equation for a conic section.

3. (+)Analyze conic sections using equations and graphs. G.GPE.3

    a. (+)Given a quadratic equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ (where B = 0), determine whether the graph is a circle, parabola, ellipse, or hyperbola G.GPE.3.A

    b. (+)Use the process of completing the square to put the equation in standard form G.GPE.3.B

    c. (+)When given a graph, be able to write the equation of the conic section, and vice versa. G.GPE.3.C

Conditional Probability and the Rules of Probability

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

9. (+)Use permutations and combinations to compute probabilities of compound events and solve problems. G.GPE.9

Using Probability to Make Decisions

Calculate expected values and use them to solve problems.

1. (+)Assign a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. S.MD.1
2. (+)Calculate the expected value of a random variable; understand that it is the mean of the probability distribution. S.MD.2
3. (+)Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; calculate the expected value. S.MD.3
4. (+)Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; calculate the expected value. S.MD.4

Use probability to evaluate outcomes of decisions.

5. (+)Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding calculating the expected values. S.MD.5
    a. (+)Calculate the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. S.MD.5.A
    b. (+)Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident. S.MD.5.B
6. (+)Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.6
7. (+)Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). S.MD.7

Polar Coordinates

Define polar coordinates and the relationship between polar coordinates and Cartesian coordinates.

1. (+)Define polar coordinates and the relationship between polar coordinates and Cartesian coordinates with and without the use of technology. PC.PC.1
2. (+)Use polar equations to model and solve problems using graphs and algebraic properties. PC.PC.2

Parametric Equations

Define parametric equations.

1. (+)Given equations for a parametric function, plot the graph and make conclusions about the geometric figure that result. PC.PE.1
2. (+)Convert between a pair of parametric equations and an equation in x and y. Model and solve problems using parametric equations. PC.PE.2

Limits

Define a continuous function.

1. (+)Determine if a function is continuous at a point. Find the types of discontinuities of a function and relate them to finding limits of a function. Use the concept of limits to describe discontinuity and end-behavior of the function. PC.L.1

Define limits.

2. (+)Demonstrate knowledge of both the definition and graphical interpretation of limits of values of functions and sequences. Verify and estimate limits using graphs, tables, and technology. PC.L.2
3. (+)Evaluate limits of functions and apply properties of limits, including one-sided limits and limits at infinity using algebra. PC.L.3

Sequences

Define sequences.

1. (+)Define arithmetic and geometric sequences and series. Model and solve word problems involving applications of sequences and series, interpret the solutions and determine whether the solutions are reasonable. PC.S.1