

# Grades 9, 10, 11, 12

Adopted 2016

## Algebra I

### Number and Quantity

The Real Number System

- B. Use properties of rational and irrational numbers [HSS-MD.B](#)
  - 3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. [N-RN.3](#)

Quantities

- B. Reason quantitatively and use units to solve problems [HSG-SRT.B](#)
  - 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. [N-Q.1](#)
  - 2. Define appropriate quantities for the purpose of descriptive modeling. [N-Q.2](#)
  - 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. [N-Q.3](#)

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## Algebra

### Seeing Structure in Expressions

- A. Interpret the structure of expressions [HSA-SSE.A](#)
  - 1. Interpret expressions that represent a quantity in terms of its context. [A-SSE.1](#)
    - a. Interpret parts of an expression, such as terms, factors, and coefficients. [A-SSE.1.A](#)
    - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE.1.B](#)
  - 2. Use the structure of an expression to identify ways to rewrite it. [A-SSE.2](#)
- B. Write expressions in equivalent forms to solve problems [HSA-SSE.B](#)
  - 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. [A-SSE.3](#)
    - a. Factor a quadratic expression to reveal the zeros of the function it defines. [A-SSE.3.A](#)
    - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. [A-SSE.3.B](#)
    - c. Use the properties of exponents to transform expressions for exponential functions. [A-SSE.3.C](#)

### Arithmetic with Polynomials and Rational Expressions

- A. Perform arithmetic operations on polynomials [HSA-APR.A](#)
  - 1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [A-APR.1](#)
- B. Understand the relationship between zeros and factors of polynomials [HSA-APR.B](#)
  - 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. [A-APR.3](#)

### Creating Equations

- A. Create equations that describe numbers or relationships [HSA-CED.A](#)
  - 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. [A-CED.1](#)
  - 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED.2](#)
  - 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. [A-CED.3](#)

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED.4](#)

### Reasoning with Equations and Inequalities

- A. Understand solving equations as a process of reasoning and explain the reasoning [HSA-REI.A](#)
  1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. [A-REI.1](#)
- B. Solve equations and inequalities in one variable [HSA-REI.B](#)
  3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. [A-REI.3](#)
  4. Solve quadratic equations in one variable. [A-REI.4](#)
    - a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form. [A-REI.4.A](#)
    - b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ . [A-REI.4.B](#)
- C. Solve systems of equations [HSA-REI.C](#)
  5. Given a system of two equations in two variables, show and explain why the sum of equivalent forms of the equations produces the same solution as the original system. [A-REI.5](#)
  6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. [A-REI.6](#)
- D. Represent and solve equations and inequalities graphically [HSA-REI.D](#)
  10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). [A-REI.10](#)
  11. Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. [A-REI.11](#)
  12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. [A-REI.12](#)

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## Functions

### Interpreting Functions

- A. Understand the concept of a function and use function notation **HSF-IF.A**
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ . **F-IF.1**
  2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F-IF.2**
  3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. **F-IF.3**
- B. Interpret functions that arise in applications in terms of the context **HSF-IF.B**
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **F-IF.4**
  5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **F-IF.5**
  6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **F-IF.6**
- C. Analyze functions using different representations **HSF-IF.C**
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-IF.7**
    - a. Graph linear and quadratic functions and show intercepts, maxima, and minima. **F-IF.7.A**
    - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **F-IF.7.B**
  8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **F-IF.8**
    - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **F-IF.8.A**
  9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **F-IF.9**

### Building Functions

- A. Build a function that models a relationship between two quantities **HSF-BF.A**
  - 1. Write a function that describes a relationship between two quantities. **F-BF.1**
    - a. Determine an explicit expression, a recursive process, or steps for calculation from a context. **F-BF.1.A**
- B. Build new functions from existing functions **HSF-BF.B**
  - 3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **F-BF.3**

#### Linear, Quadratic, and Exponential Models

- A. Construct and compare linear, quadratic, and exponential models and solve problems **HSF-LE.A**
  - 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **F-LE.1**
    - a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. **F-LE.1.A**
    - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **F-LE.1.B**
    - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **F-LE.1.C**
  - 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **F-LE.2**
  - 3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **F-LE.3**
- B. Interpret expressions for functions in terms of the situation they model **HSF-LE.B**
  - 5. Interpret the parameters in a linear or exponential function in terms of a context. **F-LE.5**

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## Statistics and Probability

### Interpreting Categorical and Quantitative Data

- A. Summarize, represent, and interpret data on a single count or measurement variable **HSS-ID.A**
    - 1. Represent data with plots on the real number line (dot plots, histograms, and box plots). **S-ID.1**
    - 2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **S-ID.2**
    - 3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). **S-ID.3**
  - B. Summarize, represent, and interpret data on two categorical and quantitative variables **HSS-ID.B**
    - 5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. **S-ID.5**
    - 6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **S-ID.6**
      - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. **S-ID.6.A**
      - b. Informally assess the fit of a function by plotting and analyzing residuals. **S-ID.6.B**
      - c. Fit a linear function for a scatter plot that suggests a linear association. **S-ID.6.C**
  - C. Interpret linear models **HSS-ID.C**
    - 7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **S-ID.7**
    - 8. Compute (using technology) and interpret the correlation coefficient of a linear fit. **S-ID.8**
    - 9. Distinguish between correlation and causation. **S-ID.9**
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## Geometry

## Geometry

### Congruence

#### A. Experiment with transformations in the plane [HSG-CO.A](#)

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. [G-CO.1](#)
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). [G-CO.2](#)
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. [G-CO.3](#)
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. [G-CO.4](#)
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. [G-CO.5](#)

#### B. Understand congruence in terms of rigid motions [HSG-CO.B](#)

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. [G-CO.6](#)
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. [G-CO.7](#)
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. [G-CO.8](#)

#### C. Prove geometric theorems [HSG-CO.C](#)

9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. [G-CO.9](#)
10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. [G-CO.10](#)
11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram

bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. [G-CO.11](#)

**D. Make geometric constructions** [HSG-CO.D](#)

- 12.** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. [G-CO.12](#)
- 13.** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [G-CO.13](#)

Similarity, Right Triangles, and Trigonometry

**A. Understand similarity in terms of similarity transformations** [HSG-SRT.A](#)

- 1.** Verify experimentally the properties of dilations given by a center and a scale factor: [G-SRT.1](#)
  - 1a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. [G-SRT.1A](#)
  - 1b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. [G-SRT.1B](#)
- 2.** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. [G-SRT.2](#)
- 3.** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. [G-SRT.3](#)

**B. Prove theorems involving similarity** [HSG-SRT.B](#)

- 4.** Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. [G-SRT.4](#)
- 5.** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. [G-SRT.5](#)

**C. Define trigonometric ratios and solve problems involving right triangles** [HSG-SRT.C](#)

- 6.** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. [G-SRT.6](#)
- 7.** Explain and use the relationship between the sine and cosine of complementary angles. [G-SRT.7](#)
- 8.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. [G-SRT.8](#)

## Circles

- A. Understand and apply theorems about circles **HSG-C.A**
  - 1. Prove that all circles are similar. **G-C.1**
  - 2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. **G-C.2**
  - 3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **G-C.3**
- B. Find arc lengths and areas of sectors of circles **HSG-C.B**
  - 5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **G-C.5**

## Expressing Geometric Properties with Equations

- A. Translate between the geometric description and the equation for a conic section **HSG-GPE.A**
  - 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **G-GPE.1**
- B. Use coordinates to prove simple geometric theorems algebraically **HSG-GPE.B**
  - 4. Use coordinates to prove simple geometric theorems algebraically. **G-GPE.4**
  - 5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **G-GPE.5**
  - 6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **G-GPE.6**
  - 7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. **G-GPE.7**

## Geometric Measurement and Dimension

- A. Explain volume formulas and use them to solve problems **HSG-GMD.A**
  - 1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. **G-GMD.1**
  - 3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. **G-GMD.3**
- B. Visualize relationships between two-dimensional and three-dimensional objects **HSG-GMD.B**
  - 4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **G-GMD.4**

## Modeling with Geometry

- A. Apply geometric concepts in modeling situations **HSG-MG.A**
    - 1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). **G-MG.1**
    - 2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). **G-MG.2**
    - 3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). **G-MG.3**
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## Algebra II

### Number and Quantity

#### The Real Number System

- A. Extend the properties of exponents to rational exponents **HSF-TF.A**
  - 1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. **N-RN.1**
  - 2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. **N-RN.2**

#### Quantities

- B. Reason quantitatively and use units to solve problems **HSG-SRT.B**
  - 2. Define appropriate quantities for the purpose of descriptive modeling. **N-Q.2**

#### The Complex Number System

- A. Perform arithmetic operations with complex numbers **HSA-APR.A**
  - 1. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real. **N-CN.1**
  - 2. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N-CN.2**
- B. Use complex numbers in polynomial identities and equations **HSA-APR.B**
  - 7. Solve quadratic equations with real coefficients that have complex solutions. **N-CN.7**

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## Algebra

### Seeing Structure in Expressions

- A. Interpret the structure of expressions [HSA-SSE.A](#)
  - 2. Use the structure of an expression to identify ways to rewrite it. [A-SSE.2](#)
- B. Write expressions in equivalent forms to solve problems [HSA-SSE.B](#)
  - 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. [A-SSE.3](#)
    - c. Use the properties of exponents to transform expressions for exponential functions. [A-SSE.3.C](#)
  - 4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. [A-SSE.4](#)

### Arithmetic with Polynomials and Rational Expressions

- B. Understand the relationship between zeros and factors of polynomials [HSA-APR.B](#)
  - 2. Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ . [A-APR.2](#)
  - 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. [A-APR.3](#)
- C. Use polynomial identities to solve problems [HSA-APR.C](#)
  - 4. Prove polynomial identities and use them to describe numerical relationships. [A-APR.4](#)
- D. Rewrite rational expressions [HSA-APR.D](#)
  - 6. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system. [A-APR.6](#)

### Creating Equations

- A. Create equations that describe numbers or relationships [HSA-CED.A](#)
  - 1. Create equations and inequalities in one variable and use them to solve problems. [A-CED.1](#)
  - 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED.2](#)
  - 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context [A-CED.3](#)

### Reasoning with Equations and Inequalities

- A. Understand solving equations as a process of reasoning and explain the reasoning [HSA-REI.A](#)
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. [A-REI.1](#)
  2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. [A-REI.2](#)
- B. Solve equations and inequalities in one variable [HSA-REI.B](#)
4. Solve quadratic equations in one variable. [A-REI.4](#)
    - b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ . [A-REI.4.B](#)
- C. Solve systems of equations [HSA-REI.C](#)
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. [A-REI.6](#)
  7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. [A-REI.7](#)
- D. Represent and solve equations and inequalities graphically [HSA-REI.D](#)
11. Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. [A-REI.11](#)

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## Functions

### Interpreting Functions

- A. Understand the concept of a function and use function notation **HSF-IF.A**
  - 3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. **F-IF.3**
- B. Interpret functions that arise in applications in terms of the context **HSF-IF.B**
  - 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F-IF.4**
  - 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **F-IF.6**
- C. Analyze functions using different representations **HSF-IF.C**
  - 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-IF.7**
    - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **F-IF.7.C**
    - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F-IF.7.E**
  - 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **F-IF.8**
    - b. Use the properties of exponents to interpret expressions for exponential functions. **F-IF.8.B**
  - 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **F-IF.9**

### Building Functions

- A. Build a function that models a relationship between two quantities **HSF-BF.A**
  - 1. Write a function that describes a relationship between two quantities. **F-BF.1**
    - a. Determine an explicit expression, a recursive process, or steps for calculation from a context. **F-BF.1.A**
    - b. Combine standard function types using arithmetic operations. **F-BF.1.B**
  - 2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **F-BF.2**
- B. Build new functions from existing functions **HSF-BF.B**
  - 3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given

the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **F-BF.3**

4. Find inverse functions. **F-BF.4**
  - a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. **F-BF.4.A**

#### Linear, Quadratic, and Exponential Models

- A. Construct and compare linear, quadratic, and exponential models and solve problems **HSF-LE.A**
  2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **F-LE.2**
  3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function **F-LE.3**
  4. For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology. **F-LE.4**
- B. Interpret expressions for functions in terms of the situation they model **HSF-LE.B**
  5. Interpret the parameters in a linear or exponential function in terms of a context. **F-LE.5**

#### Trigonometric Functions

- A. Extend the domain of trigonometric functions using the unit circle **HSF-TF.A**
  1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F-TF.1**
  2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F-TF.2**

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## Geometry

### Expressing Geometric Properties with Equations

- A. Translate between the geometric description and the equation for a conic section **HSG-GPE.A**
  2. Derive the equation of a parabola given a focus and directrix. **G-GPE.2**

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## Statistics and Probability

### Interpreting Categorical and Quantitative Data

- A. Summarize, represent, and interpret data on a single count or measurement variable **HSS-ID.A**
  - 4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **S-ID.4**
- B. Summarize, represent, and interpret data on two categorical and quantitative variables **HSS-ID.B**
  - 6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **S-ID.6**
    - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. **S-ID.6.A**

### Making Inferences and Justifying Conclusions

- A. Understand and evaluate random processes underlying statistical experiments **HSS-IC.A**
  - 1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **S-IC.1**
  - 2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. **S-IC.2**
- B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies **HSS-IC.B**
  - 3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **S-IC.3**
  - 4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **S-IC.4**
  - 5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **S-IC.5**
  - 6. Evaluate reports based on data. **S-IC.6**

### Conditional Probability and the Rules of Probability

- A. Understand independence and conditional probability and use them to interpret data **HSS-CP.A**
  - 1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **S-CP.1**
  - 2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this

characterization to determine if they are independent. [S-CP.2](#)

3. Understand the conditional probability of A given B as  $P(A \text{ and } B)/P(B)$ , and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. [S-CP.3](#)
  4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. [S-CP.4](#)
  5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. [S-CP.5](#)
- B. Use the rules of probability to compute probabilities of compound events in a uniform probability model [HSS-CP.B](#)
6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. [S-CP.6](#)
  7. Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model. [S-CP.7](#)
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## Algebra III

### Number and Quantity

- A. Explore and illustrate the characteristics and operations connecting sequences and series [HSG-GMD.A](#)
1. Express sequences and series using recursive and explicit formulas. [MS.AIII.1](#)
  2. Evaluate and apply formulas for arithmetic and geometric sequences and series. [MS.AIII.2](#)
  3. Calculate limits based on convergent and divergent series. [MS.AIII.3](#)
  4. Evaluate and apply infinite geometric series. [MS.AIII.4](#)
  5. Extend the meaning of exponents to include rational numbers. [MS.AIII.5](#)
  6. Simplify expressions with fractional exponents to include converting from radicals. [MS.AIII.6](#)
  7. Factor algebraic expressions containing fractional exponents. [MS.AIII.7](#)

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## Algebra

- A. Analyze and manipulate functions [HS.MS.A](#)
- Determine characteristics of graphs of parent functions (domain/range, increasing/decreasing intervals, intercepts, symmetry, end behavior, and asymptotic behavior). [MS.AIII.8](#)
  - Determine the end behavior of polynomial functions. [MS.AIII.9](#)
- C. Use polynomial identities to solve problems [HSA-APR.C](#)
- Prove polynomial identities and use them to describe numerical relationships. [MS.AIII.10](#)
  - Verify the Binomial Theorem by mathematical induction or by a combinatorial argument. [MS.AIII.11](#)
  - Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle. [MS.AIII.12](#)
  - Write rational expressions in simplest form. [MS.AIII.13](#)
  - Decompose a rational function into partial fractions. [MS.AIII.14](#)
  - Determine asymptotes and holes of rational functions, explain how each was found, and relate these behaviors to continuity. [MS.AIII.15](#)
- A. Perform operations on expressions, equations, inequalities and polynomials [HSN-CN.A](#)
- Add, subtract, multiply and divide rational expressions. [MS.AIII.16](#)
  - Solve polynomial and rational inequalities. Relate results to the behavior of the graphs. [MS.AIII.17](#)
  - Find the composite of two given functions and find the inverse of a given function. Extend this concept to discuss the identity function  $f(x) = x$ . [MS.AIII.18](#)
  - Simplify complex algebraic fractions (with/without variable expressions and integer exponents) to include expressing  $f(x + h) - f(x)/h$  as a single simplified fraction when  $f(x) = 1/1 - x$  for example. [MS.AIII.19](#)
  - Find the possible rational roots using the Rational Root Theorem. [MS.AIII.20](#)
  - Find the zeros of polynomial functions by synthetic division and the Factor Theorem. [MS.AIII.21](#)
  - Graph and solve quadratic inequalities. [MS.AIII.22](#)

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## Functions

- C. Analyze functions using different representations **HSF-IF.C**
  - 23. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **MS.A.III.23**
  - 24. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **MS.A.III.24**
- A. Build a function that models a relationship between two quantities **HSF-BF.A**
  - 25. Compose functions. **MS.A.III.25**
- B. Build new functions from existing functions **HSF-BF.B**
  - 26. Verify by composition that one function is the inverse of another. **MS.A.III.26**
  - 27. Read values of an inverse function from a graph or a table, given that the function has an inverse. **MS.A.III.27**
  - 28. Produce an invertible function from a non-invertible function by restricting the domain. **MS.A.III.28**
  - 29. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **MS.A.III.29**
- A. Extend the domain of trigonometric functions using the unit circle **HSF-TF.A**
  - 30. Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi-x$ ,  $\pi+x$ , and  $2\pi-x$  in terms of their values for  $x$ , where  $x$  is any real number. **MS.A.III.30**
  - 31. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **MS.A.III.31**
- B. Model periodic phenomena with trigonometric functions **HSF-TF.B**
  - 32. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **MS.A.III.32**
  - 33. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **MS.A.III.33**
  - 34. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. **MS.A.III.34**
- C. Prove and apply trigonometric identities **HSF-TF.C**
  - 35. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **MS.A.III.35**
  - 36. Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle. **MS.A.III.36**

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## Geometry

- A. Recognize, sketch, and transform graphs of functions [HSN-Q.A](#)
- 37. Graph piecewise defined functions and determine continuity or discontinuities. [MS.A.III.37](#)
  - 38. Describe the attributes of graphs and the general equations of parent functions (linear, quadratic, cubic, absolute value, rational, exponential, logarithmic, square root, cube root, and greatest integer). [MS.A.III.38](#)
  - 39. Explain the effects of changing the parameters in transformations of functions. [MS.A.III.39](#)
  - 40. Predict the shapes of graphs of exponential, logarithmic, rational, and piecewise functions, and verify the prediction with and without technology. [MS.A.III.40](#)
  - 41. Relate symmetry of the behavior of even and odd functions. [MS.A.III.41](#)
- D. Apply trigonometry to general triangles [HSG-SRT.D](#)
- 42. Derive the formula  $A = \frac{1}{2} ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. [MS.A.III.42](#)
  - 43. Prove the Laws of Sines and Cosines and use them to solve problems. [MS.A.III.43](#)
  - 44. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). [MS.A.III.44](#)

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## Statistics and Probability

- A. Explore and apply fundamental principles of probability. [HSG-GMD.A](#)
- 45. Analyze expressions in summation and factorial notation to solve problems. [MS.A.III.45](#)
  - 46. Prove statements using mathematical induction. [MS.A.III.46](#)

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## Calculus

### Number and Quantity

- A. Compute and determine the reasonableness of results in mathematical and real world situations [HSS-MD.A](#)
- 1. Estimate limits from graphs or tables. [MS.C.1](#)
  - 2. Estimate numerical derivatives from graphs or tables of data. [MS.C.2](#)
  - 3. Prove statements using mathematical induction. [MS.C.3](#)

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## Algebra

- C. Demonstrate basic knowledge of functions, including their behavior and characteristics **HSG-SRT.C**
  - 4. Predict and explain the characteristics and behavior of functions and their graphs (domain, range, increasing/decreasing intervals, intercepts, symmetry, and end behavior). **MS.C.4**
  - 5. Investigate, describe, and determine asymptotic behavior using tables, graphs, and analytical methods. **MS.C.5**
  - 6. Determine and justify the continuity and discontinuity of functions. **MS.C.6**
- C. Evaluate limits and communicate an understanding of the limiting process **HSG-SRT.C**
  - 7. Solve mathematical situations and application problems involving or using derivatives, including exponential, logarithmic, and trigonometric functions. **MS.C.7**
  - 8. Calculate limits using algebraic methods. **MS.C.8**
  - 9. Verify the behavior and direction of non-determinable limits. **MS.C.9**
- B. Use the definition and formal rules of differentiation to compute derivatives **HSN-RN.B**
  - 10. State and apply the formal definition of a derivative. **MS.C.10**
  - 11. Apply differentiation rules to sums, products, quotients, and powers of functions. **MS.C.11**
  - 12. Use the chain rule and implicit differentiation. **MS.C.12**
  - 13. Describe the relationship between differentiability and continuity. **MS.C.13**
- C. Apply derivatives to find solutions in a variety of situations **HSF-IF.C**
  - 15. Define a derivative and explain the purpose/utility of the derivative. **MS.C.15**
  - 16. Apply the derivative as a rate of change in varied contexts, including velocity, speed, and acceleration. **MS.C.16**
  - 17. Apply the derivative to find tangent lines and normal lines to given curves at given points. **MS.C.17**
  - 18. Predict and explain the relationships between functions and their derivatives. **MS.C.18**
  - 19. Model rates of change to solve related rate problems. **MS.C.19**
  - 20. Solve optimization problems. **MS.C.20**

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## Calculus

- C. Employ various integration properties and techniques to evaluate integrals **HSG-SRT.C**
  - 21. State and apply the First and Second Fundamental Theorem of Calculus. **MS.C.21**
  - 22. Apply the power rule and u-substitution to evaluate indefinite integrals. **MS.C.22**

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## Geometry

- B. Use geometric concepts to gain insights into, answer questions about, and graph various implications of differentiation **HSG-GPE.B**
  - 23. Demonstrate and explain the differences between average and instantaneous rates of change. **MS.C.23**
  - 24. Apply differentiation techniques to curve sketching **MS.C.24**
  - 25. Apply Rolle's Theorem and the Mean Value Theorem and their geometric consequences. **MS.C.25**
  - 26. Identify and apply local linear approximations. **MS.C.26**
  - 27. Analyze curves with attention to non-decreasing functions (monotonicity) and concavity. **MS.C.27**

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## Statistics and Probability

- HS-MS-C.** Adapt integration methods to model situations to problems **HS-MS-C**
  - 28. Apply integration to solve problems of area. **MS.C.28**
  - 29. Utilize integrals to model and find solutions to real-world problems such as calculating displacement and total distance traveled. **MS.C.29**
- C. Apply appropriate techniques, tools, and formulas to determine values for the definite integral **HSF-IF.C**
  - 30. Interpret the concept of definite integral as a limit of Riemann sums over equal subdivisions. **MS.C.30**

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## Integrated Mathematics

I

## Number and Quantity

Quantities

- B. Reason quantitatively and use units to solve problems **HSG-SRT.B**
  - 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.1**
  - 2. Define appropriate quantities for the purpose of descriptive modeling. **N-Q.2**
  - 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities **N-Q.3**

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## Algebra

### Seeing Structure in Expressions

- A. Interpret the structure of expressions [HSA-SSE.A](#)
  - 1. Interpret expressions that represent a quantity in terms of its context. [A-SSE.1](#)
    - a. Interpret parts of an expression, such as terms, factors, and coefficients. [A-SSE.1.A](#)
    - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE.1.B](#)
- B. Write expressions in equivalent forms to solve problems [HSA-SSE.B](#)
  - 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. [A-SSE.3](#)
    - c. Use the properties of exponents to transform expressions for exponential functions. [A-SSE.3.C](#)

### Creating Equations

- A. Create equations that describe numbers or relationships [HSA-CED.A](#)
  - 1. Create equations and inequalities in one variable and use them to solve problems. [A-CED.1](#)
  - 2. Create equations in two variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED.2](#)
  - 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. [A-CED.3](#)
  - 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED.4](#)

### Reasoning with Equations and Inequalities

- B. Solve equations and inequalities in one variable [HSA-REI.B](#)
  - 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. [A-REI.3](#)
- C. Solve systems of equations [HSA-REI.C](#)
  - 5. Given a system of two equations in two variables, show and explain why the sum of equivalent forms of the equations produces the same solution as the original system. [A-REI.5](#)
  - 6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. [A-REI.6](#)
- D. Represent and solve equations and inequalities graphically [HSA-REI.D](#)
  - 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could

be a line). **A-REI.10**

- 11.** Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, rational, absolute value and exponential functions. **A-REI.11**
- 12.** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. **A-REI.12**

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## Functions

### Interpreting Functions

- A. Understand the concept of a function and use function notation **HSF-IF.A**
  - 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ . **F-IF.1**
  - 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F-IF.2**
  - 3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. **F-IF.3**
- B. Interpret functions that arise in applications in terms of the context **HSF-IF.B**
  - 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F-IF.4**
  - 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **F-IF.5**
  - 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **F-IF.6**
- C. Analyze functions using different representations **HSF-IF.C**
  - 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-IF.7**
    - a. Graph functions (linear and quadratic) and show intercepts, maxima, and minima. **F-IF.7.A**
  - 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **F-IF.9**

### Building Functions

- A. Build a function that models a relationship between two quantities **HSF-BF.A**
  - 1. Write a function that describes a relationship between two quantities. **F-BF.1**
    - a. Determine an explicit expression, a recursive process, or steps for calculation from a context. **F-BF.1.A**
  - 2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. **F-BF.2**

### Linear, Quadratic, and Exponential Models

- A. Construct and compare linear, quadratic, and exponential models and solve problems **HSF-LE.A**
  - 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **F-LE.1**
    - a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. **F-LE.1.A**
    - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **F-LE.1.B**
    - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **F-LE.1.C**
  - 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **F-LE.2**
  - 3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **F-LE.3**
- B. Interpret expressions for functions in terms of the situation they model **HSF-LE.B**
  - 5. Interpret the parameters in a linear or exponential function in terms of a context. **F-LE.5**

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## Geometry

### Congruence

#### A. Experiment with transformations in the plane [HSG-CO.A](#)

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. [G-CO.1](#)
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). [G-CO.2](#)
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. [G-CO.3](#)
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. [G-CO.4](#)
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. [G-CO.5](#)

#### B. Understand congruence in terms of rigid motions [HSG-CO.B](#)

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. [G-CO.6](#)
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. [G-CO.7](#)
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. [G-CO.8](#)

#### C. Prove geometric theorems [HSG-CO.C](#)

9. Prove theorems about lines and angles. [G-CO.9](#)
10. Prove theorems about triangles. [G-CO.10](#)
11. Prove theorems about parallelograms. [G-CO.11](#)

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## Statistics and Probability

### Interpreting Categorical and Quantitative Data

- A. Summarize, represent, and interpret data on a single count or measurement variable **HSS-ID.A**
    - 1. Represent and analyze data with plots on the real number line (dot plots, histograms, and box plots). **S-ID.1**
    - 2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **S-ID.2**
    - 3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). **S-ID.3**
  - B. Summarize, represent, and interpret data on two categorical and quantitative variables **HSS-ID.B**
    - 5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. **S-ID.5**
    - 6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **S-ID.6**
      - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. **S-ID.6.A**
      - c. Fit a linear function for a scatter plot that suggests a linear association. **S-ID.6.C**
  - C. Interpret linear models **HSS-ID.C**
    - 7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **S-ID.7**
    - 8. Compute (using technology) and interpret the correlation coefficient of a linear fit. **S-ID.8**
    - 9. Distinguish between correlation and causation. **S-ID.9**
-

## Number and Quantity

## The Real Number System

- A. Extend the properties of exponents to rational exponents **HSF-TF.A**
1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. **N-RN.1**
  2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. **N-RN.2**
- B. Use properties of rational and irrational numbers **HSS-MD.B**
3. Explain why:
    - the sum or product of two rational numbers is rational;
    - the sum of a rational number and an irrational number is irrational;
    - and the product of a nonzero rational number and an irrational number is irrational.**N-RN.3**

## Quantities

- B. Reason quantitatively and use units to solve problems **HSG-SRT.B**
2. Define appropriate quantities for the purpose of descriptive modeling. **N-Q.2**

## The Complex Number System

- A. Perform arithmetic operations with complex numbers **HSA-APR.A**
1. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real. **N-CN.1**
  2. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N-CN.2**
- B. Use complex numbers in polynomial identities and equations **HSA-APR.B**
7. Solve quadratic equations with real coefficients that have complex solutions. **N-CN.7**

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## Algebra

### Seeing Structure in Expressions

- A. Interpret the structure of expressions **HSA-SSE.A**
  - 1. Interpret expressions that represent a quantity in terms of its context. **A-SSE.1**
    - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. **A-SSE.1.B**
  - 2. Use the structure of an expression to identify ways to rewrite it. **A-SSE.2**
- B. Write expressions in equivalent forms to solve problems **HSA-SSE.B**
  - 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **A-SSE.3**
    - a. Factor a quadratic expression to reveal the zeros of the function it defines. **A-SSE.3.A**
    - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **A-SSE.3.B**

### Arithmetic with Polynomials and Rational Expressions

- A. Perform arithmetic operations on polynomials **HSA-APR.A**
  - 1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **A-APR.1**

### Creating Equations

- A. Create equations that describe numbers or relationships **HSA-CED.A**
  - 1. Create equations and inequalities in one variable and use them to solve problems. **A-CED.1**
  - 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-CED.2**
  - 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. **A-CED.3**
  - 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. **A-CED.4**

### Reasoning with Equations and Inequalities

- A. Understand solving equations as a process of reasoning and explain the reasoning **HSA-REI.A**
  - 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **A-REI.1**

**B. Solve equations and inequalities in one variable** HSA-REI.B

**4. Solve quadratic equations in one variable.** A-REI.4

- a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form. A-REI.4.A
- b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions. A-REI.4.B

**C. Solve systems of equations** HSA-REI.C

- 6. Solve systems of linear equations algebraically, exactly, approximately, and graphically while focusing on pairs of linear equations in two variables. A-REI.6
- 7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. A-REI.7

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## Functions

### Interpreting Functions

- B.** Interpret functions that arise in applications in terms of the context **HSF-IF.B**
  - 4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F-IF.4**
  - 5.** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **F-IF.5**
  - 6.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **F-IF.6**
- C.** Analyze functions using different representations **HSF-IF.C**
  - 7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-IF.7**
    - a.** Graph linear and quadratic functions and show intercepts, maxima, and minima. **F-IF.7.A**
    - b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **F-IF.7.B**
    - e.** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F-IF.7.E**
  - 8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **F-IF.8**
    - a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **F-IF.8.A**
    - b.** Use the properties of exponents to interpret expressions for exponential functions. **F-IF.8.B**
  - 9.** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **F-IF.9**

### Building Functions

- A.** Build a function that models a relationship between two quantities **HSF-BF.A**
  - 1.** Write a function that describes a relationship between two quantities. **F-BF.1**
    - a.** Determine an explicit expression, a recursive process, or steps for calculation from a context. **F-BF.1.A**
    - b.** Combine standard function types using arithmetic operations. **F-BF.1.B**

### Build new functions from existing functions

3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **F-BF.3**
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## Geometry

### Similarity, Right Triangles, and Trigonometry

- A. Understand similarity in terms of similarity transformations **HSG-SRT.A**
  1. Verify experimentally the properties of dilations given by a center and a scale factor: **G-SRT.1**
    - a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. **G-SRT.1.A**
    - b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G-SRT.1.B**
  2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G-SRT.2**
  3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **G-SRT.3**
- B. Prove theorems using similarity **HSG-SRT.B**
  4. Prove theorems about triangles. **G-SRT.4**
  5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **G-SRT.5**
- C. Define trigonometric ratios and solve problems involving right triangles **HSG-SRT.C**
  6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G-SRT.6**
  7. Explain and use the relationship between the sine and cosine of complementary angles. **G-SRT.7**
  8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **G-SRT.8**

### Geometric Measurement and Dimension

- A. Explain volume formulas and use them to solve problems **HSG-GMD.A**
  1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. **G-GMD.1**
  3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. **G-GMD.3**

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## Statistics and Probability

### Interpreting Categorical and Quantitative Data

- B. Summarize, represent, and interpret data on two categorical and quantitative variables **HSS-ID.B**
  - 6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **S-ID.6**
    - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. **S-ID.6.A**
    - b. Informally assess the fit of a function by plotting and analyzing residuals. **S-ID.6.B**

### Conditional Probability and the Rules of Probability

- A. Understand independence and conditional probability and use them to interpret data **HSS-CP.A**
  - 1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **S-CP.1**
  - 2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **S-CP.2**
  - 3. Understand the conditional probability of A given B as  $P(A \text{ and } B)/P(B)$ , and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. **S-CP.3**
  - 4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. **S-CP.4**
  - 5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. **S-CP.5**
- B. Use the rules of probability to compute probabilities of compound events in a uniform probability model **HSS-CP.B**
  - 6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. **S-CP.6**
  - 7. Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model. **S-CP.7**

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## Integrated Mathematics III

## Number and Quantity

### Quantities

- B. Reason quantitatively and use units to solve problems **HSG-SRT.B**
  - 2. Define appropriate quantities for the purpose of descriptive modeling. **N-Q.2**

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## Algebra

### Seeing Structure in Expressions

- A. Interpret the structure of expressions [HSA-SSE.A](#)
  - 2. Use the structure of an expression to identify ways to rewrite it. [A-SSE.2](#)
- B. Write expressions in equivalent forms to solve problems [HSA-SSE.B](#)
  - 4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. [A-SSE.4](#)

### Arithmetic with Polynomials and Rational Expressions

- B. Understand the relationship between zeros and factors of polynomials [HSA-APR.B](#)
  - 2. Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ . [A-APR.2](#)
  - 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial (limit to 1st- and 2nd-degree polynomials). [A-APR.3](#)
- C. Use polynomial identities to solve problems [HSA-APR.C](#)
  - 4. Prove polynomial identities and use them to describe numerical relationships. [A-APR.4](#)
- D. Rewrite rational expressions [HSA-APR.D](#)
  - 6. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system. [A-APR.6](#)

### Creating Equations

- A. Create equations that describe numbers or relationships [HSA-CED.A](#)
  - 1. Create equations and inequalities in one variable and use them to solve problems. [A-CED.1](#)
  - 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED.2](#)

### Reasoning with Equations and Inequalities

- A. Understand solving equations as a process of reasoning and explain the reasoning [HSA-REI.A](#)
  - 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. [A-REI.1](#)
  - 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. [A-REI.2](#)

D. Represent and solve equations and inequalities graphically **HSA-REI.D**

11. Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **A-REI.11**

## Functions

### Interpreting Functions

- B. Interpret functions that arise in applications in terms of the context **HSF-IF.B**
  - 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F-IF.4**
  - 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **F-IF.6**
- C. Analyze functions using different representations **HSF-IF.C**
  - 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-IF.7**
    - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **F-IF.7.C**
    - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F-IF.7.E**
  - 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **F-IF.9**

### Building Functions

- B. Build new functions from existing functions **HSF-BF.B**
  - 3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **F-BF.3**
  - 4. Find inverse functions. **F-BF.4**
    - a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. **F-BF.4.A**

### Linear, Quadratic, and Exponential Models

- A. Construct and compare linear, quadratic, and exponential models and solve problems **HSF-LE.A**
  - 4. For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology. **F-LE.4**

### Trigonometric Functions

- A. Extend the domain of trigonometric functions using the unit circle **HSF-TF.A**
  - 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F-TF.1**

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F-TF.2**

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **F-TF.5**

Prove and apply trigonometric identities

8. Prove the Pythagorean identity  $\sin(\theta)^2 + \cos(\theta)^2 = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$ , given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle. **F-TF.8**

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## Geometry

### Congruence

#### D. Make geometric constructions [HSG-CO.D](#)

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). [G-CO.12](#)
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [G-CO.13](#)

### Circles

#### A. Understand and apply theorems about circles [HSG-C.A](#)

1. Prove that all circles are similar. [G-C.1](#)
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. [G-C.2](#)
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. [G-C.3](#)

#### B. Find arc lengths and areas of sectors of circles [HSG-C.B](#)

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. [G-C.5](#)

### Expressing Geometric Properties with Equations

#### A. Translate between the geometric description and the equation for a conic section [HSG-GPE.A](#)

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. [G-GPE.1](#)
2. Derive the equation of a parabola given a focus and directrix. [G-GPE.2](#)

#### B. Use coordinates to prove simple geometric theorems algebraically [HSG-GPE.B](#)

4. Use coordinates to prove simple geometric theorems algebraically. [G-GPE.4](#)
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). [G-GPE.5](#)
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. [G-GPE.6](#)
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. [G-GPE.7](#)

### Geometric Measurement and Dimension

**B.** Visualize relationships between two-dimensional and three-dimensional objects [HSG-GMD.B](#)

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. [G-GMD.4](#)

Modeling with Geometry

**A.** Apply geometric concepts in modeling situations [HSG-MG.A](#)

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). [G-MG.1](#)
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). [G-MG.2](#)
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). [G-MG.3](#)

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## Statistics and Probability

### Interpreting Categorical and Quantitative Data

- A. Summarize, represent, and interpret data on a single count or measurement variable **HSS-ID.A**
  - 4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **S-ID.4**
- B. Summarize, represent, and interpret data on two categorical and quantitative variables **HSS-ID.B**
  - 6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **S-ID.6**
    - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. **S-ID.6.A**
    - b. Informally assess the fit of a function by plotting and analyzing residuals. **S-ID.6.B**

### Making Inferences and Justifying Conclusions

- A. Understand and evaluate random processes underlying statistical experiments **HSS-IC.A**
    - 1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **S-IC.1**
    - 2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. **S-IC.2**
  - B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies **HSS-IC.B**
    - 3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **S-IC.3**
    - 4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **S-IC.4**
    - 5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **S-IC.5**
    - 6. Evaluate reports based on data. **S-IC.6**
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## Number and Quantity

### The Complex Number System

- A. Perform arithmetic operations with complex numbers **HSA-APR.A**
  - 3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **N-CN.3**
- D. Represent complex numbers and their operations on the complex plane **HSA-REI.D**
  - 4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **N-CN.4**
  - 5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. **N-CN.5**
  - 6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **N-CN.6**
- B. Use complex numbers in polynomial identities and equations **HSA-APR.B**
  - 8. Extend polynomial identities to the complex numbers. **N-CN.8**
  - 9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **N-CN.9**

### Vector and Matrix Quantities

- A. Represent and model with vector quantities **HSN-Q.A**
  - 1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $v$ ,  $|v|$ ,  $\|v\|$ ,  $v$ ). **N-VM.1**
  - 2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **N-VM.2**
  - 3. Solve problems involving velocity and other quantities that can be represented by vectors. **N-VM.3**
- C. Perform operations on vectors **HSN-VM.C**
  - 4. Add and subtract vectors. **N-VM.4**
    - a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **N-VM.4.A**
    - b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **N-VM.4.B**
    - c. Understand vector subtraction  $v - w$  as  $v + (-w)$ , where  $-w$  is the additive inverse of  $w$ , with the same magnitude as  $w$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips

in the appropriate order, and perform vector subtraction component-wise. [N-VM.4.C](#)

5. Multiply a vector by a scalar. [N-VM.5](#)

- a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c\langle v_x, v_y \rangle = \langle cv_x, cv_y \rangle$ . [N-VM.5.A](#)
- b. Compute the magnitude of a scalar multiple  $cv$  using  $\|cv\| = |c|v$ . Compute the direction of  $cv$  knowing that when  $|c|v \neq 0$ , the direction of  $cv$  is either along  $v$  (for  $c > 0$ ) or against  $v$  (for  $c < 0$ ). [N-VM.5.B](#)

A. Perform operations on matrices and use matrices in applications [HSN-CN.A](#)

6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. [N-VM.6](#)
7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. [N-VM.7](#)
8. Add, subtract, and multiply matrices of appropriate dimensions. [N-VM.8](#)
9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. [N-VM.9](#)
10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. [N-VM.10](#)
11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. [N-VM.11](#)
12. Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. [N-VM.12](#)

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## Algebra

### Arithmetic with Polynomials and Rational Expressions

#### C. Use polynomial identities to solve problems [HSA-APR.C](#)

5. Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle. [A-APR.5](#)

#### D. Rewrite rational expressions [HSA-APR.D](#)

7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. [A-APR.7](#)

### Reasoning with Equations and Inequalities

#### C. Solve systems of equations [HSA-REI.C](#)

8. Represent a system of linear equations as a single matrix equation in a vector variable. [A-REI.8](#)
9. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater). [A-REI.9](#)

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## Functions

### Interpreting Functions

- C. Analyze functions using different representations **HSF-IF.C**
- 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-IF.7**
- d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **F-IF.7.D**

### Building Functions

- A. Build a function that models a relationship between two quantities **HSF-BF.A**
  - 1. Write a function that describes a relationship between two quantities. **F-BF.1**
    - c. Compose functions. **F-BF.1.C**
- B. Build new functions from existing functions **HSF-BF.B**
  - 4. Find inverse functions. **F-BF.4**
    - b. Verify by composition that one function is the inverse of another. **F-BF.4.B**
    - c. Read values of an inverse function from a graph or a table, given that the function has an inverse. **F-BF.4.C**
    - d. Produce an invertible function from a non-invertible function by restricting the domain. **F-BF.4.D**
  - 5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **F-BF.5**

### Trigonometric Functions

- A. Extend the domain of trigonometric functions using the unit circle **HSF-TF.A**
  - 3. Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi - x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number. **F-TF.3**
  - 4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **F-TF.4**
- B. Model periodic phenomena with trigonometric functions **HSF-TF.B**
  - 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **F-TF.5**
  - 6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **F-TF.6**
  - 7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. **F-TF.7**
- C. Prove and apply trigonometric identities **HSF-TF.C**

9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **F-TF.9**
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## Geometry

### Similarity, Right Triangles, and Trigonometry

#### D. Apply trigonometry to general triangles **HSG-SRT.D**

9. Derive the formula  $A = \frac{1}{2} ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **G-SRT.9**
10. Prove the Laws of Sines and Cosines and use them to solve problems. **G-SRT.10**
11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **G-SRT.11**

### Circles

#### A. Understand and apply theorems about circles **HSG-C.A**

4. Construct a tangent line from a point outside a given circle to the circle. **G-C.4**

### Expressing Geometric Properties with Equations

#### A. Translate between the geometric description and the equation for a conic section **HSG-GPE.A**

3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **G-GPE.3**

### Geometric Measurement and Dimension

#### A. Explain volume formulas and use them to solve problems **HSG-GMD.A**

2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **G-GMD.2**

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## Statistics and Probability

### Conditional Probability and the Rules of Probability

- B. Use the rules of probability to compute probabilities of compound events in a uniform probability model **HSS-CP.B**
  - 8. Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model. **S-CP.8**
  - 9. Use permutations and combinations to compute probabilities of compound events and solve problems. **S-CP.9**

### Using Probability to Make Decisions

- A. Calculate expected values and use them to solve problems **HSS-MD.A**
    - 1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **S-MD.1**
    - 2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **S-MD.2**
    - 3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. **S-MD.3**
    - 4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? **S-MD.4**
  - B. Use probability to evaluate outcomes of decisions **HSS-MD.B**
    - 5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. **S-MD.5**
      - a. Find the expected payoff for a game of chance. **S-MD.5.A**
      - b. Evaluate and compare strategies on the basis of expected values. For example, **S-MD.5.B**
    - 6. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **S-MD.6**
    - 7. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). **S-MD.7**
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## Foundations of Algebra

### Equations and Inequalities

1. Interpret key features of an expression (i.e., terms, factors, and coefficients). [MS.FA.1](#)
2. Create expressions that can be modeled by a real-world context. [MS.FA.2](#)
3. Use the structure of an expression to identify ways to rewrite it. [MS.FA.3](#)
4. Simplify and evaluate numerical and algebraic expressions. [MS.FA.4](#)
5. Compare and contrast an expression and an equation and give examples of each. [MS.FA.5](#)
6. Given an equation, solve for a specified variable of degree one (i.e. isolate a variable). [MS.FA.6](#)
7. Fluently solve and check multi-step equations and inequalities with an emphasis on the distributive property, variables on both sides, and rational coefficients. Explain each step when solving a multistep equation and inequality. Justify each step using the properties of real numbers. [MS.FA.7](#)
8. Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. [MS.FA.8](#)
9. Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve inequalities of these forms fluently. [MS.FA.9](#)
10. Graph the solution point of an equation and the solution set of an inequality in one variable on a horizontal number line. For inequalities, be able to interpret and write the solution set in a variety of ways (e.g., set notation). [MS.FA.10](#)
11. Justify when linear equations in one variable will yield one solution, infinitely many solutions, or no solution. [MS.FA.11](#)

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## Functions

12. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Use function notation, where appropriate. [MS.FA.12](#)
13. Compare and contrast a function and a relation. Use appropriate strategies to assess whether a given situation represents a function or a relation (e.g., the vertical line test). [MS.FA.13](#)
14. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. [MS.FA.14](#)
15. Determine the rate of change of a linear function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Use the rate of change to determine if two lines are parallel, perpendicular, or neither. [MS.FA.15](#)
16. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. [MS.FA.16](#)
17. Create and graph the equation of a linear function given the rate of change and  $y$ -intercept. Compare and contrast up to three linear functions written in a various forms (i.e., point-slope, slope-intercept, standard form). [MS.FA.17](#)
18. Given two points, a graph, a table of values, a mapping, or a real-world context determine the linear function that models this information. Fluently convert between the point-slope, slope-intercept, and standard form of a line. [MS.FA.18](#)
19. Create and identify the parent function for linear and quadratic functions in the Coordinate Plane. [MS.FA.19](#)
20. Compare the properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [MS.FA.20](#)
21. Describe the following characteristics of linear and quadratic parent functions by inspection: domain/range, increasing/decreasing intervals, intercepts, symmetry, and asymptotic behavior. Identify each characteristic in set notation or words, where appropriate. [MS.FA.21](#)
22. Graph a system of two functions,  $f(x)$  and  $g(x)$ , on the same Coordinate Plane by hand for simple cases, and with technology for complicated cases. Explain the relationship between the point(s) of intersection and the solution to the system. Determine the solution(s) using technology, a tables of values, substitution, or successive approximations. (Limited to linear and quadratic functions only.) [MS.FA.22](#)
23. With accuracy, graph the solutions to a linear inequality in two variables as a half-plane, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes on the same Coordinate Plane. Construct graphs of linear inequalities and systems of linear inequalities without technology. Use appropriate strategies to verify points that may or may not belong to the solution set. [MS.FA.23](#)

24. Identify real-world contexts that can be modeled by a system of inequalities in two variables. (Limited to three inequalities.) [MS.FA.24](#)
  25. Identify when systems of equations and inequalities have constraints. [MS.FA.25](#)
  26. Perform simple translations on linear functions given in a variety of forms (e.g., two points, a graph, a table of values, a mapping, slope-intercept form, or standard form). Explain the impact on the parent function when the slope is greater than one or less than one and the effect of increasing/decreasing the y-intercept. [MS.FA.26](#)
  27. Given the graph of function in the form  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , or  $f(x + k)$ , where  $k$  belongs to the set of integers, identify the domain/range, increasing/decreasing intervals, intercepts, symmetry, and asymptotic behavior, where appropriate. Identify each characteristic in set notation or as an inequality, where appropriate. (Limited to linear and quadratic functions only.) [MS.FA.27](#)
  28. Identify and graph real-world contexts that can be modeled by a quadratic equation. [MS.FA.28](#)
  29. Solve quadratic equations in standard form by factoring, graphing, tables, and the Quadratic Formula. Know when the Quadratic Formula might yield complex solutions and the location of the solutions in relationship to the x-axis. Know suitable alternatives for the terminology "solution of a quadratic" and when each is appropriate to use. [MS.FA.29](#)
  30. Understand the relationship between the constants of a quadratic equation and the attributes of the graph. Recognize the relationship between the value of the discriminant and the type and number of solutions (i.e., predict the characteristics of a graph given the equation). [MS.FA.30](#)
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### Polynomials

31. Describe and identify a polynomial of degree one, two, three and four by examining a polynomial expression or a graph. [MS.FA.31](#)
32. Add and subtract polynomials using appropriate strategies (e.g. by using Algebra Tiles). [MS.FA.32](#)
33. Factor polynomials using the greatest common factor and factor quadratics that have only rational zeros. [MS.FA.33](#)
34. Justify why some polynomials are prime over the rational number system. [MS.FA.34](#)
35. Use the zeros of a polynomial to construct a rough graph of the function. [MS.FA.35](#)

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## Geometry

36. Explain and apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. [MS.FA.36](#)
37. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. [MS.FA.37](#)
38. Fluently use formulas and/or appropriate measuring tools to find length and angle measures, perimeter, area, volume, and surface area of polygons, circles, spheres, cones, cylinders, pyramids, and composite or irregular figures. Use them to solve real-world and mathematical problems. [MS.FA.38](#)
39. Solve real-world and mathematical problems involving two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. [MS.FA.39](#)

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## Statistics

40. Without technology, fluently calculate the measures of central tendency (mean, median, mode), measures of spread (range, interquartile range), and understand the impact of extreme values (outliers) on each of these values. Justify which measure is appropriate to use when describing a data set or a real-world context. [MS.FA.40](#)
41. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. [MS.FA.41](#)
42. Know when it is and is not appropriate to use a linear model to make predictions about a data set beyond a given set of values. Explain extrapolation and interpolation and the impact both have on predicted values. [MS.FA.42](#)
43. For scatter plots that suggest a linear association, informally fit a straight line and predict the equation for the line of best fit. [MS.FA.43](#)
44. Justify the relationship between the correlation coefficient and the rate of change for the line of best fit. [MS.FA.44](#)
45. Understand the difference between correlation and causation and identify real-world contexts that depict each of them. [MS.FA.45](#)